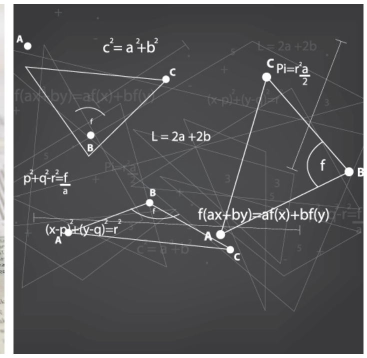
Cambridge College

SCHOOL OF EDUCATION



Math Matters

Complementary Integers

By: Dr. Arnie Good, Interim Chair
Cambridge College Mathematics Department
Arnold.Good@go.cambridgecollege.edu

Some time ago I had the need to come-up with quadratic equations having rational roots. In my search, I discovered a very simple and very interesting result. For any integer, we define its complement as its difference from the minimum power of ten that exceeds it. For example, the complement of 4 is 6, of 3 is 7, of 43 is 57 and for 103 it is 897. It turns out that both the square of the integer and its complement end in the same digits. The square of 4 is 16 and that of 6 is 36, both ending in 6. Square 3 and you get 9, square 7 and you get 49, both ending in 9. Easy enough. Squaring 43 gives us 1849 while squaring 57 yields 3249. And $103^2 = 10609$ while $897^2 = 804809$. Cool! Not deep, not profound, just cool.

Proving is just as simple. Let N be a two-digit integer. Its complement is $(100 - N)$. Squaring the complement gives us: $(100 - N)^2 = 10000 - 2N100 + N^2 = (100 - 2N)100 + N^2$. For $N < 50$ our results follow from recognizing that $(100 - 2N)100$ has no overlapping end-digits with N^2. And for $N > 50$, we replace N by $100 - N$ and prove for the complement. If it works for the complement it works for N; after all they complement each other.

But the fun's not over. Cube complementary numbers and we get complementary end digests. $43^3 = 79507$ and $57^3 = 1851893$. $07 + 93 = 100$. But proving this is not so easy. $(100 - N)^3 = 1000000 - 30000N + 300N^2 - N^3$. Tackling that successfully, well, would earn me complements!

So we turn to modular arithmetic, invented by Carl Frederick Gauss, the only mathematician I know famous enough to have all three of his names commonly known. Useful today in encryption and in verifying the accuracy of credit card numbers, it is based on the simple idea that any number can be regarded as zero. If 12 is zero, we have a clock. 3 hours after $11 = 2$ not 14. Clock arithmetic is arithmetic mod (12). If $7 = 0$, arithmetic mod (7), we have a seven-digit arithmetic: 0, 1, 2, 3, 4, 5, & 6. $4 + 4 = 1$, $4 \times 4 = 2$ (OK?), and $1/3 = 5$ because $3 \times 5 = 1$. Cute!

How does this relate to complementary integers? It relates because for, say, two-digit numbers if we set $100 = 0$ suddenly all numbers equal their last two digits. $43^3 = 79507 = 7$ and $57^3 = 1851893 = 93$. So to prove our cube-conjecture, we want to show that $N^3 + (100 - N)^3$ equals 100, that is, 0 mod (100). But here $100 - N = 0 - N = - N$. Giving us: $N - N = 0$. Got that or was I too fast?

Modular arithmetic, when the modulus is a prime number, gives us a complete arithmetic system. Suppose we work in mod (7). Here any number must equal either: 0, 1, 2, 3, 4, 5, or 6. There are additive-inverses, i.e. the additive-inverse of 3 is 4 as $3 + 4 = 0$. Every number has a multiplicative-inverse; the multiplicative-inverse of 3 is 5 as $3 \times 5 = 15 = 1$. As 7 is a *super* prime ($7 = 2{\cdot}3 + 1$ and 3 is prime), we can even find the log of any number, i.e. $\log_5(3) = 5$ as $5^5 = 3$. Factoring is even possible, even fun. Consider for example: $x^2 - 6x + 1 = (x - 2)(x + 3) = (x + 5)(x - 4)$ = and more! Buy a dime lollipop for a nickel and get 2 cents back. Hurray for mod (7)!

Beginning Fall 2016, all Math Newsletters will be distributed via the Cambridge College website.

visit us at
cambridgecollege.edu

June 2016

Introducing Cambridge College's second ever Mathematics Publication. Our hope is to share interesting articles and information on math topics for middle and high school teachers and students.

For updates on Cambridge College news, events, and academic programs, like us on Facebook and follow us on Twitter!

#mylifemycollege
#mathmatters

Cambridge College
cambridgecollege.edu

Cambridge College
SCHOOL OF EDUCATION

STEM Degree Programs Offered at Cambridge College

Master of Education in Mathematics (1-6, 5-8, or 8-12)

Initial Licensure

Credits: 33 **Credits for Licensure: 38**

The Mathematics Education program prepares students to effectively teach mathematics at the elementary (1-6), middle (5-8), and high school levels (8-12). Students learn the concepts, language, and procedures of mathematics; and develop competence in mathematics and interest in applying it to the world around them. The program builds on the College's successful student-centered curriculum that links theory and practice in a collaborative learning environment.

Master of Education in General Science (1-6 or 5-8)

Initial Licensure

Credits: 32 **Credits for Licensure: 35**

The General Science Education program provides essential science content, integrated with best practices in hands-on, inquiry-based science education. The curriculum is firmly rooted in the Massachusetts science education model with a balance of earth, life, physical and engineering sciences. Students experience a blend of seated and online content science courses.

At Cambridge College locations outside of Massachusetts, the above programs are currently non-licensure only.

STEM Certificates

Mathematics Specialist Certificate

Credits: 12

This program is best suited for current classroom teachers who wish to add mathematics as a new subject area to their professional skills and to new teacher candidates entering the teaching profession who are non-mathematics majors.

STEM Certificate

Credits: 12

The STEM Certificate in Science, Technology, Engineering and Mathematics provides early childhood, elementary, special education and middle school educators with the core background skills and content knowledge necessary to become highly-qualified mathematics, science and engineering educators serving our younger students. The courses combine math, science and engineering content with methodology at the elementary and middle school levels. The 12 graduate credits include both seated and online courses, making for a very user friendly learning experience.

For more information on the School of Education's STEM Degree and Certificate Programs, contact admissions at 1-800-829-4723.

The Teacher Education Program at Cambridge College is awarded TEAC accreditation by the Inquiry Brief Commission of the Council for the Accreditation of Educator Preparation (CAEP) for a period of five years, from May 2014-May 2019. The accreditation does not include individual education courses that the EPP offers to P-12 educators for professional development, re-licensure, or other purposes.