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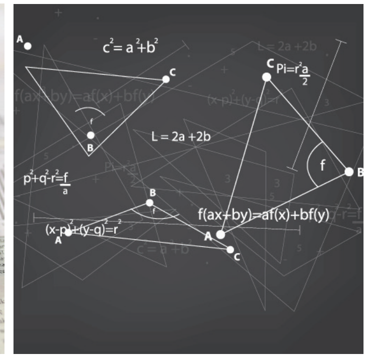

Slide & Divide

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It's called Slide and Divide, and according to a student of mine who just blurts this phrase out in the middle of one of my lectures, it will solve every quadratic equation. I say, 'no way' but ask him to tell me more after class. If he's right, I promise him an extra five points on the next quiz. I know this will never happen because when I get him alone, I'll surprise him with $\pi x^2 + ex + \varphi = 0$ (where φ is the Golden Ratio). But it turns out, I'm the one surprised, and he's earned the five points. Slide and Divide always works. Well at least up until a few centuries ago. So how does it work, why does it work, and what made it stop working. We begin with an easy example:

Consider $3x^2 + 14x + 11 = 0$. First the ***slide***: We move the lead-coefficient, 3, over to the constant term, 11, ignoring the coefficient of x, 14, all together, giving us $x^2 + 14x + 33 = 0$. This factors easily into $(x + 3)(x + 11)$ giving us $x = -3$ and -11. Next we ***divide***, meaning we divide both $x = -3$ and $x = -11$ by our slider value of 3, writing $x = -1$ & $-11/3$. While the math may be questionable, the answer is not. Nevertheless, when we look at the general quadratic we see some hope for this technique. What we are doing is replacing $ax^2 + bx + c = 0$ by $x^2 + bx + ac = 0$, both of which have the same discriminant $b^2 - 4ac$. So Slide and Divide is not entirely without merit. Turns-out our only oversight is that we've skipped a few steps.

This is what's going on: First we got rid of the lead coefficient replacing $3x^2 + 14x + 11 = 0$ with $x^2 + 14/3x + 11/3 = 0$. Next we set $x = t/3$ (where 3 was our lead coefficient) giving us $t^2/9 + 14/9\,t + 11/3 = 0$. Multiplying by 9 yields $t^2 + 14t + 33 = 0$, which is our "slide." But $x = t/3$ which is our "divide." Slide and Divide is no more than a change in variables without actually changing the variable: Sly and Hide?

But there is so much more. Turns-out, Slide and Divide is all that's left of a very general technique used many years ago to solve every, yes every, polynomial equation. Solving polynomial equations was the fad in the mid-1600s and for about a century thereafter. Solving, in many cases, meant finding rational solutions as both imaginary solutions and irrational solutions were neither understood nor even, for some, acceptable. (Over one hundred years would pass until Cauchy and Gauss made complex numbers respectable and another one hundred years still until Dedekind and Cantor did the same for irrational.) So how were polynomial equations solved? Let's look at one.

$$3x^3 + (11/2)\,x^2 + 3x + 1/2 = 0$$

Step 1 is to divide-out the lead coefficient: $x^3 + (11/6)x^2 + x + 1/6 = 0$

Step 2 is a change-of-variables: Let $x = t/k$ **getting:** $(1/k^3)\,t^3 + (11/6k^2)\,t^2 + (1/k)\,t + 1/6 = 0$

Step 3 is multiplying through by k^3 **giving us:** $t^3 + (11/6)\,kt^2 + k^2t + (1/6)\,k^3 = 0$

The trick is to choose k in order to knockout all the values in the denominators. In our example, this requires that $k = 6$, as our goal is a polynomial equation whose lead coefficient is 1 and all of whose other coefficients are integers. Why? Because it is not too hard to convince ourselves that any rational solution to a polynomial equation whose lead coefficient is 1 and all of whose other coefficients are integers must itself be an integer. Nor is it hard to convince ourselves that if the lead coefficient is 1 then any integer solution must divide the constant term. All that remains to solving our cubic is a series of guess-and-checks best accomplished using synthetic division. In the case of our particular example, which turns out to be: $t^3 + 11t^2 + 36t + 36 = 0$, $t = -2, -3$ & -6 from which it quickly follows that $x = -1/3$, $-1/2$ & -1. Had no rational solution been found, a rational 17th Century mathematician might have concluded no solution exists and call it a day. Slide and Divide always worked even if our 17th Century mathematician didn't!

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