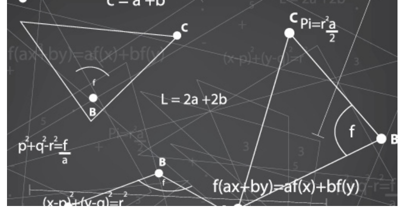

Cambridge College Mathematics Newsletter

The Rule of 24

December 2015

Here's something you probably didn't know: The square of any prime number more than 3 (trust me, quite a few are still left) minus one is always divisible by 24. 25 – 1 =24, 49 – 1 = 2x24, 121 – 1= 5x24, 169 – 1 = 7x24. There's no stopping it; I call it the 24 Divisor Rule. A friend of mine who is an electrical engineer checked out the first 1000 primes on an Excel spreadsheet and found no exceptions. I wasn't surprised. But in mathematics, you prove things in general as even one million confirmations mean zilch. Unfortunately, he also found some not-ready-for -prime-time numbers for which the rule also applied, such as 25 whose square minus 1 = 624 = 26x24. The proof is at the end of this article. Until then we'll keep you in suspense. But if *this* rule doesn't weed-out only primes, is there one that does?

One of the better rules for determining which numbers are prime requires a $5 calculator and knowledge about square roots. This is the rule Gauss used in his attempt to find all prime numbers less than 1,000,000. And he had no $5 calculator – but he did have a $1,000,000 brain. Here's the rule: If N is composite, N can always be divided by a prime number less than its square root. For example, is 149 prime? Well the square root of 149 = 12.206… Smaller primes are: 2, 3, 5, 7, and 11. (one is not prime; one "is a lonely number" and primes get too much attention these days to be lonely). By using our inexpensive calculator, we easily confirm neither 7 nor 11 divide 149 and we have handy division rules to rule out 2, 3, and 5.

But if we have a rule to rule out primes, do we have one to find primes? For big primes, the ones you patent, no; but small ones, suitable for a high school/middle school classroom, yes. Let $P(n) = n^2 + n + 1$. P(n) generates prime numbers for n up to 41. Check it out for yourself. So why would an ordinary person need so many prime numbers? Think Halloween. Forget the candy; it's not good for kids anyway. Give out prime numbers. They're inexpensive, will not cause cavities, and if lost or stolen, easily replaced. Can you just see the look on their happy faces?

OK, the proof you've been waiting for: Let P be a prime number greater than 3. $P^2 – 1 = (p – 1)(p + 1)$. As 3 must divide one of any three consecutive numbers, 3 must divide (p – 1), p, or (p + 1). But it can't divide p, so it must divide (p – 1) or (p + 1). But p = 2k + 1 for some value of k. Hence: $(p – 1)(p + 1) = (2k)(2k + 2) = 4k(k + 1)$. So 4 divides $p^2 – 1$. Finally, either k or k + 1 must itself be divisible by 2, meaning 8 divides $p^2 – 1$. As 8 and 3 are relatively prime, 3x8 = 24 divides $p^2 – 1$ QED!

We hope you enjoyed this article, which is the first of many more to come in future editions of the Cambridge College Mathematics Newsletter.

Enjoy the holidays and your winter break!

Future editions of the Mathematics Newsletter will be solely electronic.

Cambridge College is planning to launch the Achievement Institute: Closing the Achievement Gap Teaching Math to Struggling Students

Keep an eye out!

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